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The Attitude Toward Probabilities of Portfolio Managers: an Experimental Study*

Nicolas Roux

Paris School of Economics

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Abstract

This paper proposes an experiment about the attitude toward probabilities on a population of portfolio managers. Its aim is to check whether or not portfolio managers are neutral toward probabilities. Meanwhile, it presents a experimental protocole that highlights an inconsistency between two experimental techniques. It also introduces a new functional form for the probability weighting function. Results unambiguously show that portfolio managers are not neutral toward probabilities and that they display a strong heterogeneity in their preferences.

Abstract

Cet article decrit une expérience visant à étudier l'attitude face aux probabilités d'une population de gérants de portefeuille. Son but est de vérifier si cette population est neutre face aux probabilités. Le protocole expérimental utilisé permet par ailleurs de mettre en valeur une incohérence entre deux techniques de révélation des préférences. Une nouvelle fonction de pondération des probabilités est également introduite. Les résultats montrent clairement que les gérants de portefeuille déforment autant les probabilités que les sujets "classiques" participant aux expériences.

1 Introduction

Expected Utility Theory (henceforth EUT) is by far the most widely used descriptive theory of choice under risk in economics. Since Allais and his famous paradoxes, it has been recognized that EUT could not replicate choices in some situations. In order to account for the observed systematic deviations from the main theory, generalizations of EUT were proposed, notably Prospect Theory by Kahneman and Tversky (1979) and its "rank dependent form" Rank Dependent Expected Utility Theory (henceforth RDEU) by Quiggin (1982). One of their innovations is the introduction of what I will call an *attitude toward probabilities*. Deviations from EUT were explained by the fact that, unlike what is supposed in EUT, people did not treat probabilities linearly, and this was modelled by the so-called *probability weighting function*. Since then, the shape of this function has been extensively studied in experiments and the general properties of the attitude toward probabilities are by now pretty well known. This paper relates a new experimental investigation on that topic. Its contributions are two fold: one is experimental, two are methodological.

On the methodological side, a first innovation consists in the specification of the model being used (a RDEU model). Many functional forms have been proposed to capture the attitude toward probabilities. Some are one-parameter functions (Prelec(1998)), others are two-parameter functions like Wu and Gonzales (1999) or Lattimore et al. (1992). So far, the parameters of all of the two-parameter functional forms governed the shape of the function in a similar way, that is one parameter captured the level of distortion and the other one captured the level of elevation. As suggested by Isaac Meilijson¹, I introduce a new two-parameter weighting function, the cumulative distribution function of the beta law, whose advantage is that its shape is governed in a different manner. One parameter captures the degree of overweighting of small probabilities and the other one captures the degree of underweighting of large probabilities. The function thus allows to observe the correlation between those two fundamental properties of the attitude toward probabilities. The correlation over our sample is up to 80%. I do not think such an effect has ever been highlighted.

The second methodological contribution relates to the particular design of the experiment. A major shortcoming of experiments is the lack of data they allow to get from one individual in one session. As suggested by Jean-Christophe Vergnaud, a protocole was designed so as to improve the amount of information extracted from one individual in a given number of questions. The principle of this design is to take into account the information about an individual that is obtained at the beginning of the experiment so as to subsequently calibrate each question in a way that improves the information it provides. To do so, two elicitation procedures were used, namely the Trade-off method and the Certainty Equivalent method, that turned out to be inconsistent with each other, meaning that they made one individual reveal different preferences. As a result, the original aim of the design could not be fully reached, but this inconsistency raised very interesting issues about the job done by experiments.

¹Isaac Meilijson from the university of Tel Aviv gave me the idea of using the cumulative distribution function of the beta law as a probability weighting function

The empirical one relies on the interpretation that should be given to the concept of attitude toward probabilities. The view carried on by Prospect Theory is that non-linearities in the treatment of probabilities is a characteristic of human behavior that must be considered as preferences, as is the non-linear treatment of outcomes. It is thus an innate property of human behavior (deriving from some evolutionary process). In other words, people are perfectly aware of the situation and behave in such a way simply because they "like" it. However, since expected utility remains the rational way of dealing with risk, one might claim that non-linearities in the treatment of probabilities in experiments only reflect some sort of "naive" behavior. People would not behave that way if they had a better knowledge or experience of what they are doing. So people might be able to learn the "efficient" behavior. For instance, according to Plott (1996), people have expected utility preferences but "these underlying preferences only surface in choice environment where subjects are given sufficient opportunities and incentives for deliberation and learning" (Gus Van De Kuilen (2007)). Gus Van De Kuilen (2007) tested experimentally this hypothesis by asking subjects to make a series of choices with ongoing feedbacks, that is they could observe the resolution of uncertainty after each choice. He found that subjects in this setting exhibit a convergence to linearities in their treatment of probabilities, thus showing that experience of a particular risky decision makes the decision maker converge to EUT. However, those results only suggest that people are expected utility maximizers in the particular setting where they take repeated decisions and where they observe a resolution of uncertainty after each choice. Now, many economic decisions cannot be thought of as being represented by such a frame. Indeed, people often face one-shot decisions without substantial experience about the nature of the underlying risk. So studying decisions where people do not frequently observe the consequence of their decision remains important. In such a context, is there any place for some kind of learning? It is hardly contestable that people learn how to generally behave under risk as they get an experience of it. So, when they get this experience at dealing with risk, do they tend to behave as expected utility maximizers in one-shot decisions? Such an issue has been studied by Harbaugh et al. (2002): they linked the attitude toward probabilities of the respondent to his age (on a sample going from 5 to 64 years old subjects) and found that the tendency to transform probabilities tends to diminish with age, suggesting that general experience of risky decisions makes the attitude toward probabilities tend to neutrality, even without ongoing feedbacks. Fox et al. (1996) took a different approach: they ran an experiment on a sample of option traders, people who are trained at managing risk and selected for their ability to do it. They found that the median subject of their sample was an expected utility maximizer under risk, whereas the median subject of population of students showed a non-linear treatment of probabilities. This also suggests that people converge to neutrality as they get aware of what they are doing. I will adopt a similar approach here by running an experiment on a sample of portfolio managers.

Should we expect portfolio managers to be more neutral toward probabilities than anyone else? Portfolio managers are supposed to handle their clients' financial assets. In order to define the best constitution of a portfolio, they have to collect information about the economic environment so as to evaluate the risks associated with different assets. Then, they optimize their investments with respect to the "taste" of their clients. For instance, if a client asks for a

high returns portfolio at the cost of a high risk, the portfolio manager has to compose the "best" portfolio of this type. Therefore, those people deal with uncertainty everyday, since part of their job is to evaluate the risk (the true probabilities) associated with assets. They can be viewed as dealing with risk when they pick the best assets. However, they do not decide with their own preferences under risk but with their clients' preferences. Yet, it is reasonable to think that they have a great level of expertise at deciding under risk. Therefore, if one thinks that non-linearities in the treatment of probabilities are a mistake characterizing the lack of experience of decision making under risk, then one must expect a population like portfolio managers to be close to neutrality toward probabilities (at least closer than people who are not trained at dealing with risk). An experiment was therefore run in order to find out whether the portfolio manager display a neutral attitude toward probabilities. While results show that portfolio managers are more consistent than "usual subjects" in the sense that their answers fit closer to a given model of decision, results clearly show that they do not treat probabilities linearly, and even suggest that they are not closer to linearity than "usual subjects" .

The paper will be organized as follows: in the first section, I explain the basic concepts of the models used in decision under risk. I define what the attitude toward probabilities is. Then, I present the specification being used. I introduce a new functional form for the probability weighting function. The third part is devoted to the experiment and the inconsistencies it helped highlight. The last section covers the results.

2 Introduction to the descriptive analysis of decision under risk

I shall describe here the basic concepts of the descriptive analysis of decision under risk, emphasizing that of attitude toward probabilities. A decision is taken under risk if the decision maker does not know the consequence of his decision with certainty. However, by opposition to uncertainty, he knows all of the possible events as well as their objective probabilities. Such a decision can be seen as choosing one among a set of lotteries, where a lottery is defined as a distribution of probabilities over a set of outcomes. Therefore, risky decisions are often studied by making people choose between lotteries involving monetary outcomes. The aim of the descriptive analysis of decision under risk is then to be able to replicate people's choices between lotteries. EUT states that choices between lotteries can be accounted for by comparing their expected utility. For instance, $(x, p; 0, 1 - p) \succ (x', p'; 0, 1 - p')$ if and only if $pv(x) > p'v(x')$ with $v(0) = 0$, $(x, p; 0, 1 - p)$ being the lottery that gives a probability p of winning x euros and the probability $1 - p$ of winning 0 ². $v(\cdot)$ is called the value function and is unique up to a increasing affine transformation. It can be seen as representing an attitude toward outcomes. People do not treat outcomes linearly in their evaluation of the utility of a lottery.

However, as first demonstrated by Allais (1953), EUT can not explain people's choices in some particular contexts. I present an exemple highlighting one

²I will only use lotteries of this type in the paper

intuitive shortcoming of EUT. Consider the lottery:

$$(1000000, p; 0, 1 - p).$$

How much would you pay for it? If your behavior under risk is well described by EUT, then the maximum price at which you can buy the lottery is given by its certainty equivalent, defined as:

$$v(ce) = pv(1000000)$$

hence,

$$ce = 1000000v^{-1}(p)$$

Now if one assumes that $v(\cdot)$ is a concave function then the certainty equivalent is a convex function of p . This implies that the higher the probability of winning, the stronger the effect of a given variation of probability on the certainty equivalent. For example, increasing the probability of winning from 0.4 to 0.5 has more impact on the certainty equivalent than increasing it from 0 to 0.1, and it has a lower impact than increasing it from 0.9 to 1. Whereas the second claim sounds intuitive, the first one does not. Indeed, many readers would pay more for getting a possibility of winning than for increasing their probability of winning from 0.4 to 0.5. This pattern of behavior consisting of paying more for an increase of probability occurring at the extremes of the probability range cannot be accounted for by EUT unless one considers value functions convex for small gains and concave for big gains for instance. Since the behavioral pattern we have seen seems to exist for any level of gains, this explanation does not hold.

A solution to this puzzle consists in considering that people do not treat probabilities linearly in the evaluation of the utility of a lottery. Such a solution was proposed by Kahneman and Tversky (1979). In their famous Prospect Theory, the evaluation of the utility of a lottery involved not only an attitude toward outcomes, but also an attitude toward probabilities represented by a function called the *probability weighting function*. Formally, $(x, p; 0, 1 - p) \succ (x', p'; 0, 1 - p')$ if and only if $w(p)v(x) > w(p')v(x')$ where $w(\cdot)$ is the probability weighting function. It maps the probability range into $[0, 1]$ with $w(0) = 0$, $w(1) = 1$ and $w(\cdot)$ strictly increasing. This function thus associates to any objective probability a *decision weight*, that is to say a transformation of the probability that will be used in the computation of the utility of the lottery. Note that those weights are not the result of a bias in the evaluation of probabilities, since objective probabilities are known under risk. The way an individual transform probabilities in Prospect Theory must rather be seen as preferences under risk, as is the transformation of outcomes.

The structure of attitude toward probabilities was extensively studied by experiments. The results highlighted three properties for the probability weighting function representing a common structure of attitude toward probabilities. I use Prelec's terminology to present these properties.

1. Over/Underweighting

The decision weight associated to a small probability is generally higher than this probability while the decision weight associated to a medium or

large probability is generally lower than the probability. This property is necessary to generate the observed pattern of risk behavior, that is risk seeking for small probabilities and risk aversion for medium and large probabilities.

2. Decreasing Relative Sensitivity (also called subproportionality)

This property derives from the observation of the so-called common ratio effect. The common ratio effect is a particular behavior widely observed when people are facing two choices, each between two lotteries calibrated in a special way (it was first set out by Allais (1953)). For instance, when an individual is facing the following choices:

$$(300, 0.9) \text{ vs } 100 \text{ and } (300, 0.09) \text{ vs } (100, 0.1)$$

he is very likely to choose the 100 euros in the first choice and the lottery (300, 0.09) in the second one. Expressing those choices within the framework of Prospect Theory yields the two inequalities:

$$w(0.9)v(300) < v(100) \text{ and } w(0.09)v(300) > w(0.1)v(100)$$

or equivalently,

$$w(0.9) < \frac{v(100)}{v(300)} < \frac{w(0.09)}{w(0.1)}$$

On this example, we can see that decreasing both probabilities by 90%, make the associated weights diminish in different proportions. More precisely, the weight of the lower probability has diminished from a lower proportion than the weight of the higher probability, thus making the agent prefer the lottery involving 300 euros once the probabilities of both lotteries have been decreased by 90 percent. Since this behavior was observed for the whole range of probabilities, the common ratio effect was turned into a general property of the attitude toward probabilities called the diminishing relative sensitivity that can now be defined as: decreasing two probabilities in the same proportions make the weight of the lower probability decrease in a lower proportion than the weight of the higher probability.

3. Increasing absolute sensitivity near $p = 0$ and $p = 1$

This last property of $w(\cdot)$ states that a given variation of probabilities will have a stronger impact on the utility of a lottery if it occurs near the endpoints of the range of probabilities than in the middle of it. Our first example was an illustration of that principle. The implication for the shape of the probability weighting function is that it must be concave for small probabilities and convex for large ones with an inflexion point somewhere in the middle of the interval $[0,1]$.

To sum up, Prospect Theory stated that Expected Utility Theory needed to be generalized by taking the attitude toward probabilities into account and that this could be done by replacing probabilities by decision weights in the weighted average of outcomes evaluating the utility of a lottery.

"Improvements" of the theory were proposed afterwards, notably because Prospect Theory obliged the agent to violate first order dominance on some special choices. Rank Dependent Expected Utility Theory (henceforth RDEUT) by Quiggin (1982) was the first theory tackling this problem while maintaining the main characteristics of Prospect Theory. I do not further explain the differences between Prospect Theory and RDEUT since both theories are equivalent on the type of lotteries I have used in the experiment. I will consider for the rest of the paper that I work within the RDEU framework.

3 The specification

In this section, I fully describe the model I have chosen to estimate knowing that the theory will be the RDEU theory. As stated in the first section, a RDEU decision maker is characterized by two functions: the value function $v(\cdot)$ and the probability weighting function $w(\cdot)$.

The value function will be a power function, $v(x) = x^\lambda$. Three functions are often retained in the experimental literature: the power function, the exponential function, and the expo-power function. The most widely used in the experimental literature is the power function. However, there is no consensus about which one of them should be used. Wakker and Tversky (1993) and Kahneman and Tversky (1992) showed that the power function was normatively attractive. However, from the empirical point of view, it is for now impossible to claim that one fits better than the other two.

As suggested informally by Isaac Meilijson, the probability weighting function will be represented by the cumulative distribution function of the beta law (henceforth betacdf): $\text{betacdf}(p, \alpha, \beta)$. Since it is the first time that such a specification is used, let me introduce its characteristics. First, it respects all of the three properties described in the first part, namely the over/underweighting of small/large probabilities, the decreasing relative sensitivity and the increasing absolute sensitivity near 0 and 1. Second it is a two parameter function (the parameters are α and β). According to our data, it seems more relevant to represent the attitude toward probabilities with a two parameter function than with a one parameter function like the one introduced by Prelec (1998) for instance. In effect, since our aim is here to describe the attitude toward probabilities of each individual, a one parameter function would not be flexible enough to fit the data. Third it fits the data as well as other existing two parameter functions like those introduced by Prelec (1998) or Wu and Gonzales (1999). What is interesting about this function is the way in which the two parameters govern its shape. To explain this last statement, let me recall how previous functional forms work. In many of them, one parameter governs the degree of curvature of the function and the other one governs its elevation. Wu and Gonzales (1999) associated psychophysical characteristics with each of the parameters of their function. The parameter governing curvature is seen as representing "discriminability" while the one governing elevation is seen as representing "attractiveness" (figures 1 and 2).

The new function presented here, the betacdf, is governed in the way shown in figure 3 and 4. One parameter governs the intensity of overweighting of small probabilities (α) and the other one governs the intensity of underweighting of

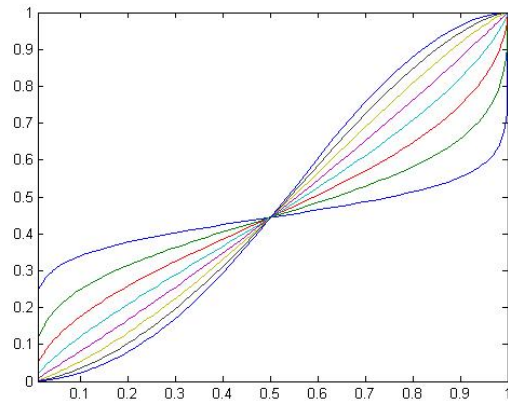


Figure 1: The shape of the probability weighting function of Wu and Gonzales (1999) with respect to the parameter governing curvature.

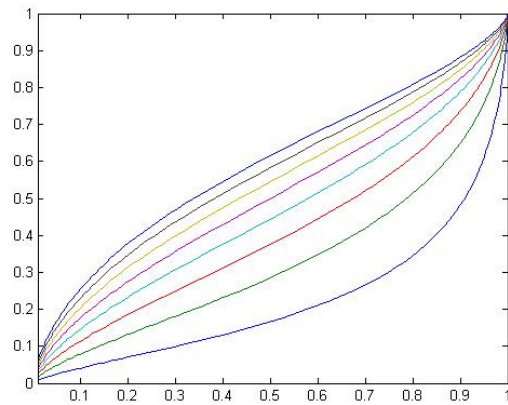


Figure 2: The shape of the probability weighting function of Wu and Gonzales (1999) with respect to the parameter governing elevation.

large probabilities (β). More precisely, when both parameters equal 1, the function is linear. The closer to 0 α and β , the higher the degree of respectively overweighting of small probabilities and underweighting of large probabilities. Therefore, the betacdf reproduces the inverse S shaped probability weighting function when α and β are smaller than one, as shown in figure 5. On the contrary, if both parameters are higher than 1, the function will show underweighting of small probabilities and overweighting of large probabilities. Note that the point at which the curve crosses the diagonal depends on the relative value of both parameters. For instance, if α is smaller than β then the curve crosses the diagonal at $p > 0.5$. This represents the fact that the subject overweights small probabilities more than he underweights large probabilities. If both parameters are equal then the diagonal is crossed at $p = 0.5$. We can try to give an interpretation to those parameters. α and β can be viewed as representing the intensity of absolute sensitivity near respectively 0 and 1. They therefore show how strong will be the possibility and certainty effects on an individual's choices. A possible interest of this function is that the correlation between the two parameters represents the correlation between absolute sensitivity near 0 and absolute sensitivity near 1. I do not think it has been shown that those characteristics were related.

The stochastic model relating the theory to observed binary choices will be a Fechner model of random errors (Hey and Orme(1994)). I could have chosen two other stochastic models, namely the Tremble model from (Harless and Camerer (1994)) and the Random Utility model from (Loomes and Sugden (1995)), but the Fechner model has proved to be superior at fitting the data (see Wilcox 2007(b)). I will further assume that the errors are normally distributed and homoskedastic ³. I will explain more precisely how the stochastic model is used in the last section.

Now I have precised the theoretical framework I have chosen, I shall move on to the way data were collected.

4 The experiment

The experiment was conducted at the Laboratoire d'Economie Experimentale de Paris 1 (LEEP) under the supervision of Michèle Cohen, Maxim Frolov and Jean-Marc Tallon. We got 89 "usual subjects" of the LEEP and 16 portfolio managers from the asset management firm *PMA gestion*. As previously stated, this study is about the attitude toward probabilities of portfolio managers. The group of "usual subjects" will therefore be used to compare results.

A few weeks before the experiment, the group of portfolio managers received a lecture about the field of decision under risk and uncertainty so as to familiarize themselves with the concepts and especially the experimental tools being used.

The experiment was not only about decision under risk. There were a few questions about decision under ambiguity afterwards. The subjects were all introduced to the aim of the experiment, as well as its rules during around 20 minutes. They were specially briefed about the incentive scheme, so as to make sure that they were convinced to be "playing" for real money. The gains went from 5 euros to 120 euros. The average payment was around 30 euros. From

³The latter assumption implies that I do not take into account recent advances about the way the noise can be modeled (see Blatavsky (2007))

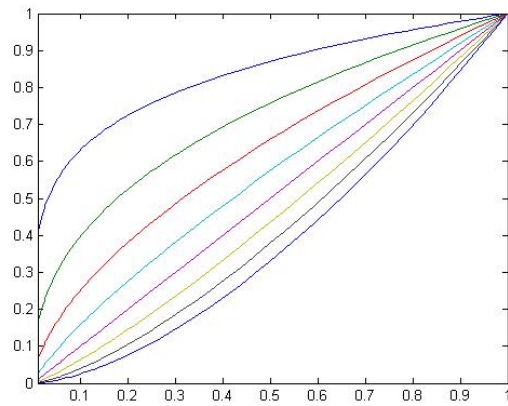


Figure 3: The shape of the betacdf with respect to the parameter governing the degree of overweighting of small probabilities

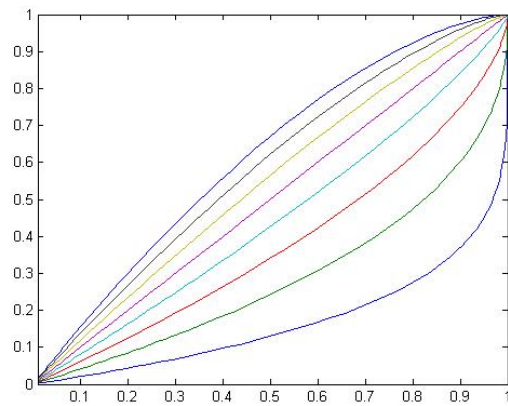


Figure 4: The shape of the betacdf with respect to the parameter governing the degree of underweighting of large probabilities

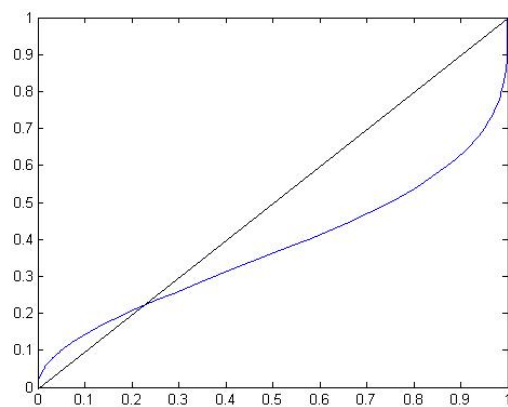


Figure 5: The betacdf when $\alpha = 0.5$ and $\beta = 0.3$

a student's point of view, such an expected payoff is definitely worth thinking things through carefully. From an asset manager's point of view, it is probably not. However, they had a powerful incentive to answer as if it were important: curiosity. Indeed, they were told during the lecture a few weeks before the experiment that they would get their own results afterwards, and that those results would be studied to explain behaviors on financial markets. This is the reason why I am confident that the data they delivered would not have been more reliable with greater monetary incentives.

In the part about decision under risk, each subject was asked around 60 questions. Answering this part took the respondents 7 minutes on average. I now precisely describe the protocole of the experiment since its particular design lead us to interesting insights about some experimental tools.

4.1 The protocole

It was originally designed so as to improve the amount of information extracted from one individual in a given number of questions, because a major shortcoming of experiments is the small number of questions that can be asked to a subject in one session. Indeed, an individual is likely to get tired (bored) before having answered 100 binary choices, so that asking him too many questions would make him provide "noisy" answers. A way to improve the amount of information obtained from a given number of questions is to ask "good questions". For instance, it is useless to ask an individual who has previously shown a strong risk aversion for a given probability of winning to choose between a lottery involving this same probability of winning and its mathematical expectation. Indeed, the answer can be predicted by the experimentalist almost with certainty. In that case, a good way of proceeding to avoid asking useless questions is to take our previous information about the subject's behavior into account to calibrate the question in a smart way. Here, this would mean asking him to choose between the lottery and some amount of money smaller than the mathematical expectation. Therefore the principle of the protocole is to get a first idea about the individual's preferences (assuming that he is RDEU maximizer) on a first set of questions and subsequently to use this information to ask only "smart" questions. The protocole is thus divided in three parts:

1. In the first part, the value function is elicited.
2. The second part determines the probability weighting function
3. At the beginning of the third part, we have our first idea about the way the individual behaves under risk since we have roughly determined his preferences in a RDEU model. The third part is then composed of "smart" questions.

The first part uses the so-called Trade-off method (TO method) from Wakker and Deneffe (1996). It is a way of calibrating questions asked to a subject so as to make him reveal his value function independently of his probability weighting function. It is based on the following property of the RDEU model: if an individual shows indifference between the lotteries:

$$(x_0, p; R, 1 - p) \text{ and } (x_1, p; r, 1 - p)$$

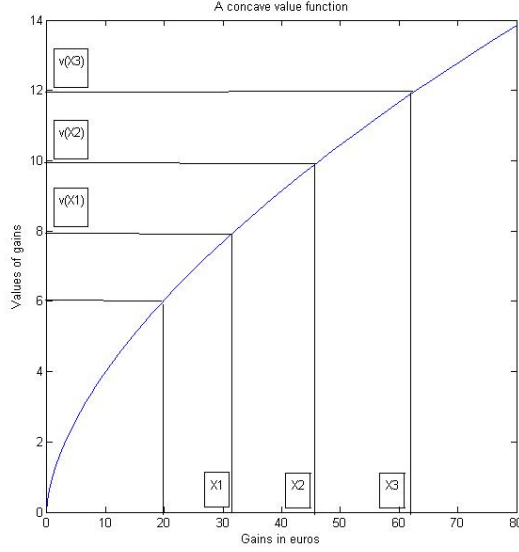


Figure 6: Examples of values elicited from the Trade-off methods for an individual with a concave value function

as well as between the lotteries:

$$(x_1, p; R, 1 - p) \text{ and } (x_2, p; r, 1 - p)$$

where $x_0 > R > r$, then a few calculations within the framework of the RDEU model show that:

$$v(x_1) - v(x_0) = v(x_2) - v(x_1)$$

In other words, these preferences imply that x_0 , x_1 and x_2 are equally spaced in term of utility for the individual.

In practice, the researcher fixes x_0 , p , r and R and makes the subject reveal the x_1 that makes him indifferent between the first two lotteries by a series of binary choices (I used the so-called bisection method to determine the x_1 . I will explain later what this consists in). In our case, $x_0 = 20$, $p = 0.5$, $r = 0$ and $R = 10$. Once I had a series of gains x_0 , x_1 , x_2 , x_3 equally spaced in terms of utility, I used a non linear least squares approximation to get the value of the parameter of the weighting function. Figure 6 shows how should be distributed the values x_1 , x_2 and x_3 relative to each other in the case of a concave value function.

The second part of the protocole, in which the probability weighting function is elicited, was based on the CE method: this method is simpler than the previous one since it just consists in making the subject reveal his certainty equivalent for a given lottery by a series of binary choices between the lottery and some certain amounts of money (again using the bisection method).

How was this information used to determine the shape of the probability weighting function?

Consider the following lottery:

$$(x, p; 0, 1 - p)$$

By setting $v(0) = 0$, the rank dependent expected utility of this lottery is:

$$V_{RDEU}(L) = w(p)v(x)$$

The certainty equivalent ce of the lottery is defined as:

$$v(ce) = w(p)v(x)$$

so,

$$w(p) = \frac{v(x)}{v(ce)}.$$

With our specification of the value function,

$$w(p) = \left(\frac{x}{ce}\right)^\lambda$$

The researcher sets the values of p and x and the subject reveals ce . Given that the value of λ , the parameter of the value function, is estimated from the first part, every certainty equivalent observed directly indicates the transformation of the probability p through the probability weighting function (assuming that the estimated value of lambda is the right one).

I therefore used this property to roughly determine the shape of the probability weighting function, by making the subject reveal his certainty equivalents for the lotteries: $(x, p; 0, 1 - p)$ with $p = 0.1, 0.5, 0.9$ and $x = 50, 80$ euros. The shape was then determined with 6 points. Actually, these 6 points provided two evaluations with three points each of the probability weighting function: one for 50 euros and one for 80 euros, so that I could check the consistency of the elicited decision weights. Figure 7 shows an example of those two evaluations of the probability weighting function for one individual. This individual clearly exhibits a "classic" inverse S shaped weighting function.

At this step, note that some individuals did not provide answers consistent enough to allow evaluate their preferences. I therefore had to class the individuals by group, depending on how well their preferences could be identified:

- Group 1 contains the subjects whose preferences could be completely evaluated, that is to say I got an estimation of both their value function and their probability weighting function.
- Group 2 is composed of people for which only the value function could be estimated. Their answers in the second part did not allow to clearly identify some profile of the probability weighting function. More precisely, the individual exhibited different attitudes toward probabilities when the gain was 50 euros and when it was 80 euros. Figure 8 shows the elicited probability weighting function of an individual in group 2. As you can see, this individual overweighted the probability 0.5 when the gain was 50 euros and he underweighted it when the gain was 80 euros.

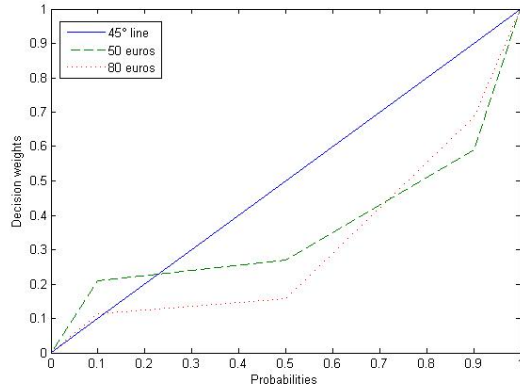


Figure 7: Evaluation of the probability weighting function, based on the first 6 certainty equivalents elicited during the second step of the protocole

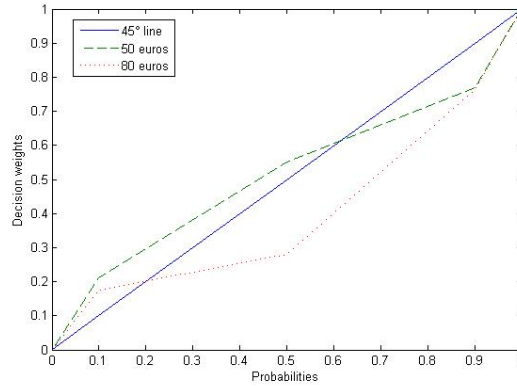


Figure 8: Evaluation of the probability weighting function, based on the first 6 certainty equivalents elicited during the second step of the protocole, for an individual from group 2

- Group 3 is composed of people who could not be identified at all. I will explain what happened with those individuals after having described the bisection technique.

Once the outliers were removed the composition of groups was the following: there were 42 subjects in group 1 (and 11 portfolio managers), 10 individuals in group 2 (2 portfolio managers), and 38 individuals in group 3 (2 portfolio managers).

The third part, in which questions were adapted to the individual's known behavior was also based on the CE method with the same type of lotteries. I added 9 other certainty equivalents corresponding to the lotteries involving $p = 0.3, 0.7$ and $x = 50, 80$ euros and to the lotteries involving $p = 0.1, 0.3, 0.5, 0.7, 0.9$ and $x = 20$ euros.⁴ As in the first two parts, the technique for making the individual reveal his certainty equivalent using a series of binary choices was the bisection method. I need to explain what this method does in order to go further. The researcher wants to know, say the certainty equivalent of the lottery $(x, p; 0, p)$. He is going to fix minimum and maximum values for this certainty equivalent: ce_{min}, ce_{max} . Then he asks the subject to choose between the lottery and the middle of the interval $[ce_{min}, ce_{max}]$, ce_{cent} . If the subject picks the lottery, then his certainty equivalent lies within the interval $[ce_{cent}, ce_{max}]$ (on the contrary, if he prefers the amount of money, then his certainty equivalent is within the interval $[ce_{min}, ce_{cent}]$). Therefore, those two values now represent the minimum and maximum levels for the certainty equivalent. The subject can be asked again to choose between the lottery and the middle of this new interval. By repeating the same operation a few times, the researcher gets a small interval in which relies the certainty equivalent. For instance, with the lottery $(80, 0.5, 0, 0.5)$, I repeated the operation 4 times (that is I asked the individual to make 4 binary choices), and I got the certainty equivalent within one of the following intervals: $[0, 5]$, $[5, 10]$, ..., $[70, 75]$, $[75, 80]$.

Let me now explain how I managed to calibrate questions. I adjusted the center of the interval $[ce_{min}, ce_{max}]$. Indeed, once the preferences of an individual were evaluated, at the end of the first two steps, I had a prior on his certainty equivalent for any new lottery. I used this prior to fix ce_{cent} at the level of this anticipated certainty equivalent for all subsequent lotteries. Moreover, I reduced the width of the interval $[ce_{min}, ce_{max}]$. The individual had thus no obvious choices to make, since he was directly close to his "switching point". Therefore, if the first evaluation was right, I could precisely determine the individual's certainty equivalent by asking fewer questions.

Of course, I could apply this procedure on the individuals in group 1 since I had a first approximation of their preferences. However, I could not apply with much precision the procedure on group 2 and even more on group 3, that is to say people had to answer some obvious questions during the whole experiment.

This way of proceeding has a shortcoming: consider the case where the individual's choices indicate that the certainty equivalent is contained in one of the extreme intervals. With the lottery $(80, 0.5; 0, 0.5)$, if the answers lead

⁴I elicited the certainty equivalents of the same lotteries for all subjects (whatever the group in which they were attached), but depending on their degree of identification, this elicitation was more or less precise. Of course, the more I knew about the preferences after the first two steps, the more precise was the elicitation of the certainty equivalent.

to the interval $[75, 80]$, the only conclusion we can make is that the certainty equivalent is higher than 75 euros but we cannot put an upper bound on it. Of course, in that example, it does not matter since the certainty equivalent cannot be expected to be higher than 80 euros. On the contrary, one can easily see that this is going to be an issue if I reduce substantially the width of the interval. In that case, if the certainty equivalent is in the "top" interval, it is likely that it is in fact higher than what we elicited. As we will see later, this indeed prevented me from getting with certainty some of the 15 certainty equivalents for most of the individuals.

I can now go back to the existence of group 3, the group of people who were not identified at all. The parameter of their value function could not be estimated in the first part (using the Trade-off method) and they were then considered as totally unknown (I could not identify the attitude toward probabilities either, because I would have needed an estimation of the value function in order to estimate the probability weighting function). How come the protocole failed estimating their value function? The reason is related to the shortcoming I just explained. The bisection method was used to elicit the values x_1 , x_2 and x_3 . Many people exhibited a value of x_3 in the "top interval" so that I did not know whether I had the right value. Since the estimation of the parameter of the value function was entirely based on the three values, I considered that it was not known.

To sum up, I had for each individual at the end of the experiment:

- one first estimation of the parameter of the value function from step 1 (TO method)
- 15 certainty equivalents coming from around 50 binary choices (CE methods)
- the group in which the individual was classed, that is the extent to which his profile could be identified

Now the protocole has been described, I can move to a first unexpected result, the inconsistency of the two elicitation methods (TO and CE).

4.2 The inconsistency between the TO and CE methods

Another problem arised from the design of the experiment. The identification of preferences operated in the first two parts of the protocole did not work so well because the two elicitation methods being used (CE and TO methods) turned out to be inconsistent with each other. More precisely, I estimated the whole model on the binary choices I got from the last two parts (where the CE method was used). I then compared the values of the parameter of the value function obtained from this estimation (based on the CE method) to the value of the same parameter obtained in the first part (using the TO method). In table 1, I took both values of the same parameter for all the individuals in the population of the usual subjects of the LEEP and in the population of the portfolio managers, and I computed their means in both populations as well as the correlation between those two different evaluations of the same parameter.

The results show that both ways of estimating do not provide the same values. Indeed, the correlation should have been close to 1 if they did. Even more

	Usual Subjects	Portfolio Managers
Mean(λ_{CE})	0.57	0.68
Mean(λ_{TO})	0.92	0.92
Corr($\lambda_{CE}, \lambda_{TO}$)	-0.16	-0.26

Table 1: Consistency of the estimations of the parameter of the value function from the Trade-off method and the Certainty Equivalent method

interesting, the mean of this parameter from the CE method is significantly lower than the mean from the Trade-off method.

How can we interpret such a result? Note first that one can find previous works showing such an inconsistency: Abdellaoui (2000) found a mean of the parameter λ up to 0.9 using the Trade-off method, while Wu and Gonzales (1999) found a mean of 0.49 using the CE method. However I could not find any paper noticing and explaining this issue. I will therefore propose some possible interpretations that would be worth being tested.

The interpretation can raise questions about either the experimental methods or the way of modeling decision making under risk.

On the one hand, one can solve the problem by claiming that one of the two methods does not provide relevant behaviors, so that the preferences revealed are not the true preferences under risk. More precisely, the argument could be stated as follows: the task asked in the Trade-off method is too difficult in the sense that subjects cannot really comprehend a choice between two lotteries, unlike the CE method in which individuals can more easily tell whether or not they would buy a lottery for a given price.

On the other hand, if one thinks that both experimental tasks provide a relevant behavior (that is worth being studied), then the model itself must be questioned. One might say that we just have not found the right model and that we shall seek for new one able to rationalize the observed behaviors based on unique preferences. However, it could also be claimed that the problem relates more generally to the way economists model decision under risk. Indeed, it is reasonable to think that the observed discrepancy comes from the fact that people use two different cognitive processes to make decision in the two different frames, resulting in different preferences when behaviors are grasped through a RDEU model. This kind of explanation is bad for economics because it implies that preferences are actually governed by the frame in which the choice is presented. An individual cannot be seen anymore as having one single preference ordering over lotteries. From that point of view then, it would be more relevant to modelize directly the cognitive process being used.

All this is pure speculation but I think it is a problem that is worth being studied. For now, I will not use the data from the TO method, I will rather focus on the answers from the CE method, that is around 50 binary choices.

5 The results

5.1 The econometric tools

In order to estimate the model, I used the binary choices made by each individual.⁵ Therefore, for each individual I had a sample of around 50 binary responses. Each choice the individual made can be modelled within the RDEU framework in the following way:

$$(g, p; 0, 1 - p) \succ c$$

where c is a certain amount of money, is equivalent to

$$w(p)v(g) > v(c)$$

The stochastic model retained, the Fechner model of homoskestatic random errors, associates the distance between the utilities of the two options ($w(p)v(g) - v(c)$) with a probability of choosing the first one. It does so by applying on this distance an increasing function Φ that maps \mathbb{R} into $[0, 1]$, with $\Phi(0) = 0.5$ and $\Phi(x) = 1 - \Phi(-x)$. In our case, this function will be the cumulative distribution function of the normal law. Taking a Fechner model with this function is the same as assuming that there is an error in the process of evaluation of the utility of an option and that this error is normally distributed with variance equal to 1. In that case, the Fechner model is a probit model. Note that the latent variable is the distance of utility between the two options, which is a non-linear form of the parameters. The log-likelihood associated with some observed set of choices can be written as:

$$\sum_{i=1}^L (y_i \log(\Phi(w(p_i)v(g_i) - v(c_i))) + (1 - y_i) \log(1 - \Phi(w(p_i)v(g_i) - v(c_i))))$$

where L is the number of choices the individual had to make, y_i takes the value 1 if the lottery is chosen and the value 0 if the sure amount is picked. Each question i involves different values p_i , c_i and g_i .

To estimate the model, I maximized this function with respect to the three parameters α , β and λ for each individual using the *maxlik* procedure of the econometrics toolbox of Matlab 7b. The same estimation was performed by Maxim Frolov using the NLP procedure of SAS. This procedure also provided estimations of the standard deviations of the estimators. Those standard deviations were also computed by bootstrap so as to check the reliability of the estimations provided by SAS. The results provided by both methods were indeed consistent. I then used the standard deviations coming from SAS. The estimated coefficients of the portfolio managers' models and the corresponding standard deviations of the estimators are shown in table 2. I also computed confidence intervals (at a level of 10%) assuming that the estimators were normally distributed. Again, the relevance of this assumption was confirmed by the results from bootstrap. The confidence intervals for the portfolio manager's models are shown in table 3.

Besides of these econometric tools, I use graphics of the probability weighting function to easily visualize the results. This representation is obtained as the one described in the second part of the protocole. I briefly recall how I proceed to

⁵I could not use the certainty equivalent because some certainty equivalents were not known for sure

get it. We have 15 certainty equivalents corresponding to 15 lotteries involving the outcomes 20, 50 and 80 euros and the probabilities 0.1, 0.3, 0.5, 0.7 and 0.9. Since the lotteries I use are all of the type $(x, p; 0, 1 - p)$, the following equation holds:

$$w(p) = \left(\frac{ce}{x}\right)^\lambda$$

where λ is the parameter of the value function. Its value will now be the one estimated by the maximum likelihood procedure.

Given this value of λ , each observed certainty equivalent indicates the decision weight associated to the probability p . We therefore have three different representations of the probability weighting function: one for each outcome. And each of those representations is composed of five points, corresponding to the five probabilities involved. This allows checking whether the individual has the same attitude toward probabilities for every gain and more generally whether he is consistent in his choices. I also recall that some of the certainty equivalents elicited could not be the right ones, due to the problem described in the section devoted to the experiment. Some certainty equivalents might be higher (lower) than the number we have gotten. Since the decision weight is computed using directly the elicited value of the certainty equivalent, it will suffer from the same uncertainty about its true value. If the true certainty equivalent is possibly higher (lower) than the elicited value, the corresponding decision weight will also be possibly higher (lower) than what is displayed. I therefore indicate on the graph representing the probability weighting function which ones of the decision weights could be actually higher or lower than what is showed: an arrow pointing down (up) indicates that the decision weight might be lower (higher). I have plotted the three curves for each portfolio managers on figure 8. The blue one represents the evaluation of the weighting function when the gain is up to 20 euros. The green and red ones respectively concern the gains of 50 and 80 euros. The fourth curve (the smooth one) is the parametric weighting function with the parameters α and β estimated by the maximum likelihood procedure.

Results are divided in four parts: the first one is about the new specification for the probability weighting function; the second one is about the individual results about the attitude toward probabilities of portfolio managers; the third one is devoted to the aggregated results. An interpretation is given in the fourth part.

5.2 Results about the new specification

As I previously explained, the betacdf fits the data in the same way as preceeding two-parameter functional forms. Its contribution then relies in the correlation it allows to compute. The correlation between the two parameters α and β can be seen as representing the link between the degree of increasing sensitivity near 0 and the degree of increasing sensitivity near 1. I have plotted estimated values of α and β over the whole population in figure 3: it shows a strong positive correlation up to 80%. This result suggests that people who are very "sensitive" at one side of the probability range are also sensitive at the other side.

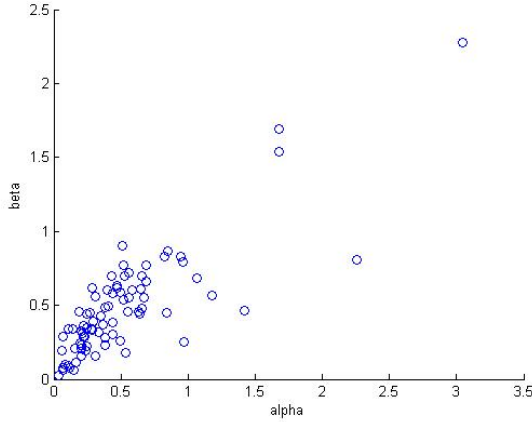


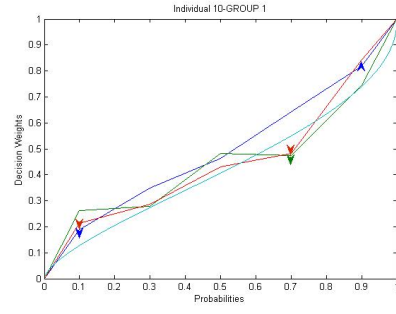
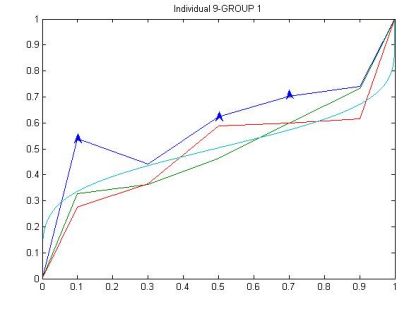
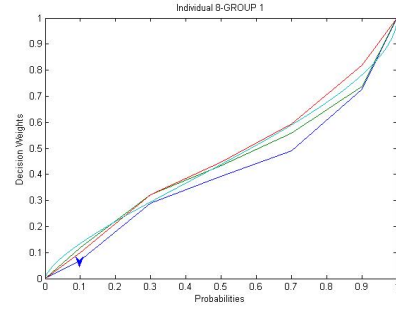
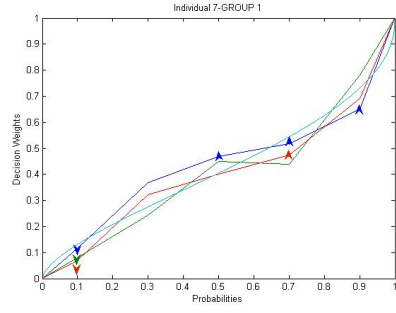
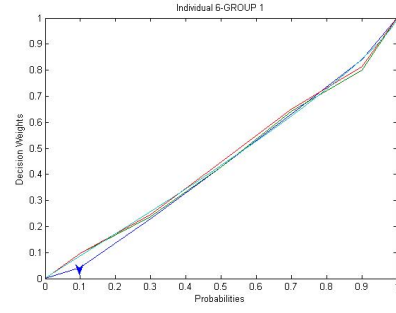
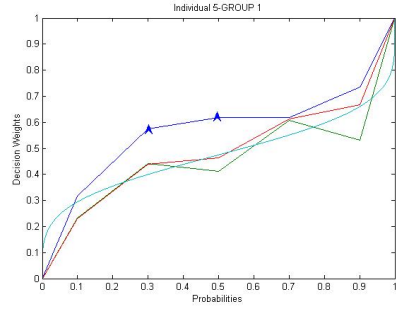
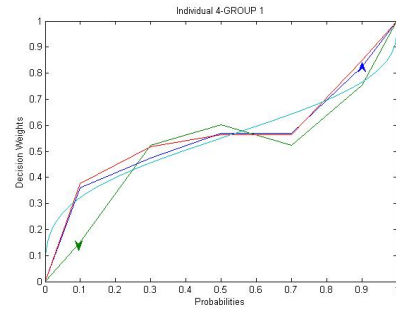
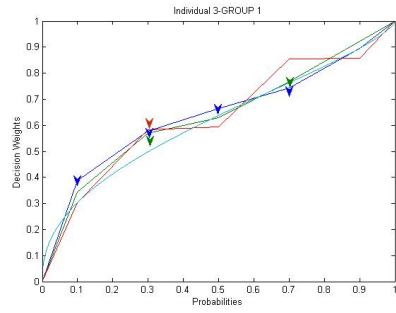
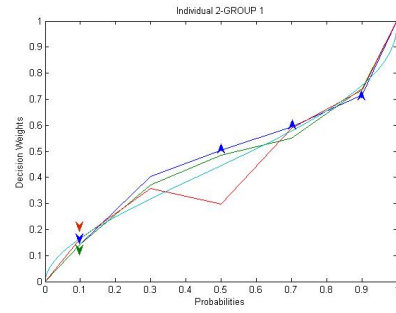
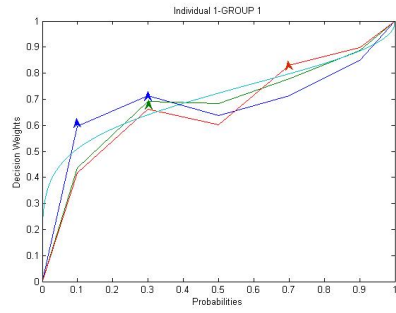
Figure 9: Couples of estimated (α, β) over the whole population

5.3 Individual Results

The first striking result of the study concerns the "quality" of the data provided by the portfolio managers. Indeed, the comparison between the curves of portfolio managers and the rest of the population reveals that the three curves of portfolio managers are generally much closer to each other than the three curves of normal subjects. To see this, I have also plotted the curves of ten representative individuals from the LEEP's sample in figure 9. This result seems consistent with the fact that portfolio managers have a great expertise at handling decision making under risk. Even if some certainty equivalents might be under/overestimated, I think the picture remains quite clear. Each portfolio managers unambiguously shows a certain pattern of distortion of probabilities, and each one of them seems to fit this pattern pretty closely.

Second, from the same graphics it seems obvious that the portfolio managers are not all neutral toward probabilities. All the portfolio managers distort significantly the objective probabilities, that is the confidence interval shown in table 3 of at least one of the two parameters of the weighting function does not contain 1. Actually, almost all of them have parameters α and β significantly lower than 1 (except for individual 6 whose α may be equal to one). So, almost all of them exhibit the classic inverse S shape of the probability weighting function. Except for the individual 6 they all have a concave value function. In other words, they respect the principle of diminishing sensitivity for both outcomes and probabilities. At the individual level then, it is hardly contestable that portfolio managers distort probabilities.

The third interesting result concerns the diversity of attitude toward probabilities in the sample of portfolio managers. As I said, they all show the classic pattern of attitude toward probabilities, nevertheless, they are very different from each other in the degree of over/underweighting of probabilities. This shows that pooling the data without checking them at the individual level would cause a real loss of information. Moreover, this shows that a one parameter functional form like that introduced by Prelec (1998) is not flexible enough to



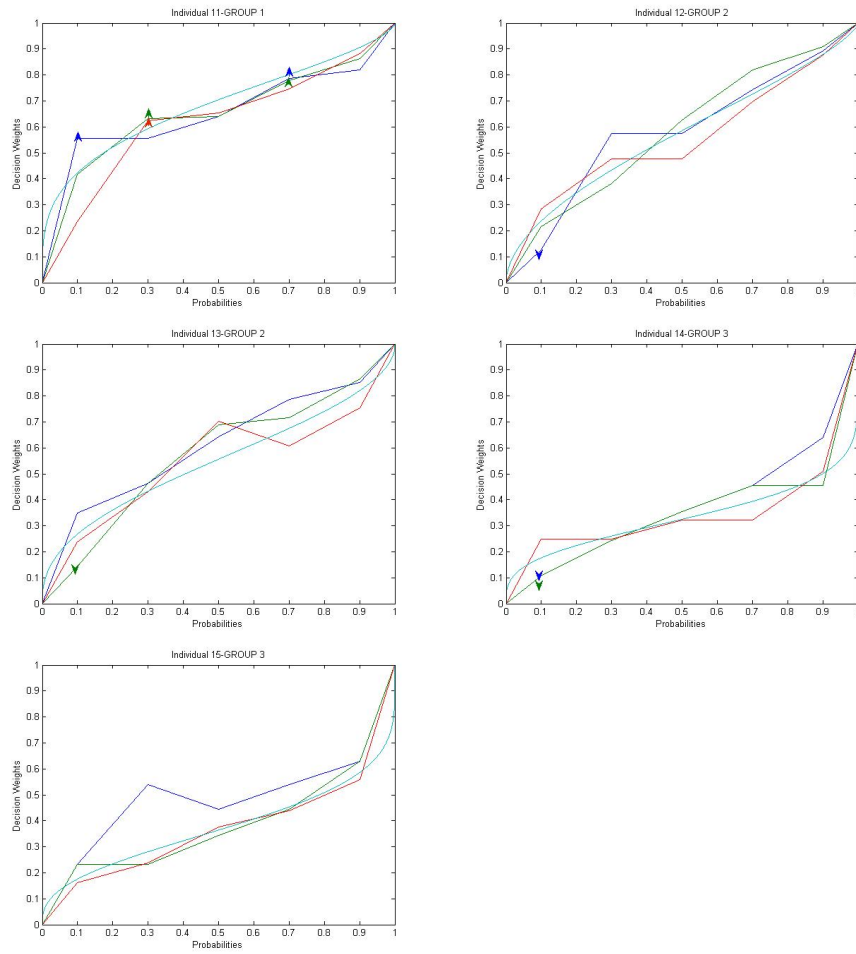


Figure 10: The elicited attitude toward probabilities for every portfolio manager

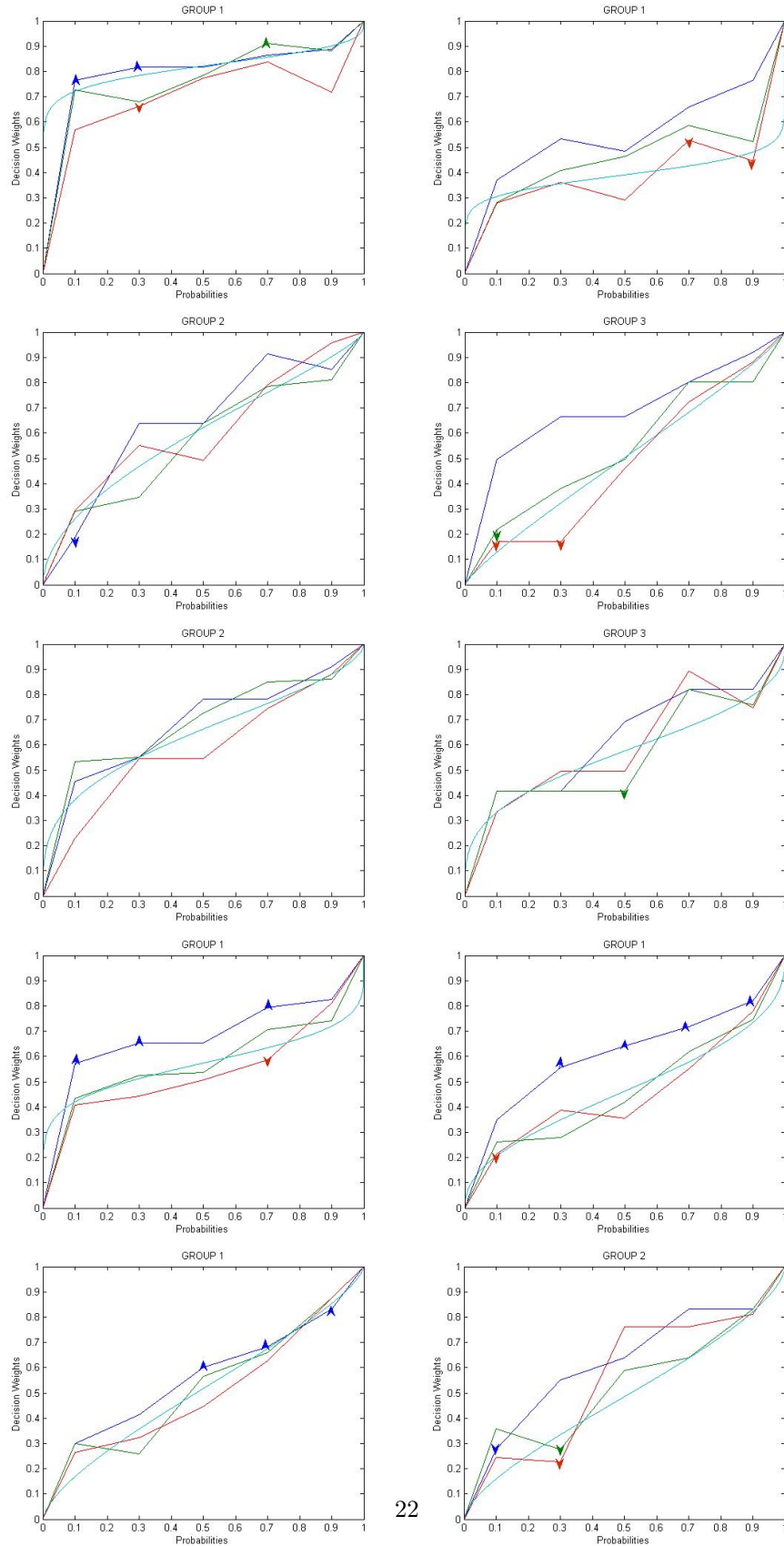


Figure 11: The elicited attitude toward probabilities of a subject from the LEEP

Coefficeint	alpha(std)	beta(std)	lambda(std)
indiv1	0.19 (0.07)	0.46 (0.12)	0.56 (0.08)
indiv 2	0.55 (0.09)	0.46 (0.07)	0.65 (0.06)
indiv 3	0.43 (0.07)	0.7 (0.10)	0.69 (0.06)
indiv 4	0.28 (0.08)	0.34 (0.09)	0.54 (0.05)
indiv 5	0.25 (0.05)	0.22 (0.04)	0.60 (0.06)
indiv 6	0.96 (0.08)	0.79 (0.03)	1.06 (0.08)
indiv 7	0.63 (0.08)	0.46 (0.06)	0.72 (0.07)
indiv 8	0.68 (0.07)	0.55 (0.03)	0.89 (0.07)
indiv 9	0.21 (0.06)	0.21 (0.05)	0.59 (0.06)
indiv10	0.65 (0.19)	0.48 (0.10)	0.56 (0.07)
indiv11	0.29 (0.08)	0.62 (0.19)	0.56 (0.09)
indiv12	0.53 (0.06)	0.70 (0.03)	0.69 (0.08)
indiv13	0.41 (0.08)	0.5 (0.10)	0.56 (0.05)
indiv14	0.31 (0.07)	0.16 (0.08)	0.75 (0.07)
indiv 15	0.38 (0.06)	0.23 (0.03)	0.77 (0.07)

Table 2: Estimated coefficients (with the associated standard error) of the model for every portfolio manager

Confidence Interval	alpha	beta	lambda
indiv 1	[0.07,0.30]	[0.25,0.67]	[0.42,0.69]
indiv2	[0.38 , 0.71]	[0.33, 0.58]	[0.53, 0.76]
indiv3	[0.30, 0.55]	[0.53,0.86]	[0.59,0.78]
indiv4	[0.14,0.41]	[0.18 ,0.49]	[0.44, 0.71]
indiv5	[0.15 ,0.34]	[0.13,0.30]	[0.48,0.63]
indiv6	[0.81,1.10]	[0.73, 0.84]	[0.91,1.20]
indiv7	[0.48,0.77]	[0.36,0.55]	[0.60,0.83]
indiv8	[0.55,0.80]	[0.48,0.61]	[0.76, 1.01]
indiv9	[0.10, 0.31]	[0.12, 0.29]	[0.47, 0.70]
indiv10	[0.32, 0.97]	[0.31,0.64]	[0.43,0.68]
indiv11	[0.15, 0.42]	[0.30,0.93]	[0.39,0.72]
indiv12	[0.42, 0.63]	[0.64,0.75]	[0.55, 0.82]
indiv13	[0.26, 0.55]	[0.32,0.67]	[0.46,0.65]
indiv14	[0.18,0.43]	[0.02,0.29]	[0.63, 0.86]
indiv15	[0.26 , 0.49]	[0.17, 0.28]	[0.64,0.89]

Table 3: Confidence intervals associated with the estimated coefficients of Table 2 at a level of 10%

capture the attitude toward probabilities at the individual level, since it obliges the probability weighting function to cross the diagonal at a given point ($1/e$).

I can now move on to the study of aggregated result.

5.4 Aggregated Results

Results on aggregated data are also provided in order to compare my study to that from Fox et al. (1996). since their study consisted in an experiment about decision under risk on a sample of options traders, people who are to be expected to have a great aptitude at dealing with decision making under risk, as are the portfolio managers. The technique used to elicit preferences is also similar to what is done in the present experiment. They proceeded in two steps: the first one was supposed to elicit the value function (and was based on the same principle as the Trade-off method), the second one elicited decision weights (using the certainty equivalent method). However, they did not use the bisection method. They did not even use binary choices. Rather, they directly asked the value of interest to the subject. For example, when they wanted to elicit a certainty equivalent, they directly asked the subject to provide the maximum price at which he would buy the lottery.

Although both the population of interest and the elicitation technique can be seen as pretty similar, the way Fox et al. (1996) treated the data is quite different from what I did up to now.

First, they found in their first step that 80% of the subjects displayed a linear value function. Furthermore, the median individual also exhibited linearity in the treatment of outcomes. Unlike what I did, they kept those results and used them to elicit the decision weights.

Moreover, the elicited decision weights were aggregated, meaning that Fox et al. (1996) took, for each probability, the median decision weight. They thus constructed the median probability weighting function of their sample (considering that the value function was linear). The median attitude toward probability was thus found to be neutral, that is the function was linear. Since, they found in previous studies on students that the median probability weighting function had the classic inverse S shape, they concluded that those results must be taken as evidence that option traders are expected utility maximizers.

In order to check whether their results are consistent with ours, I computed for each probability (0.1, 0.3, 0.5, 0.7 and 0.9) and each gain (20, 50 and 80 euros) the median decision weight. Figures 12 to 16 show the resulting median probability weighting functions. In the first two figures, you can find the three median probability weighting functions (one for each gain) for each of the two populations. Note that the functions associated with 50 and 80 euros are quite close to each other, suggesting that people have the same attitude toward probabilities for both gains. However, the probability weighting function is a bit higher when the gain is 20 euros for both populations. One can claim that there is a threshold between 20 euros and the two other gains. When gains are only up to 20 euros, people may take their decisions as they were playing because the amount at stake are insignificant. (Bear in mind that some individual decision weights may be different from the ones used to compute the median. I do not think this is very relevant at the aggregated level since these errors at the individual level seem to have canceled out when pooling the data. For example,

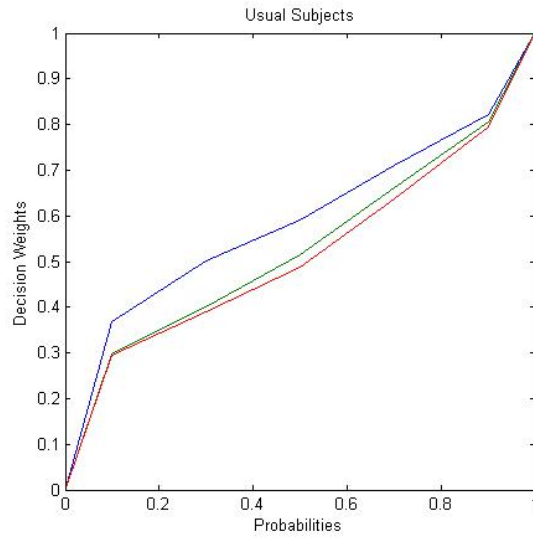


Figure 12: The median probability weighting functions of usual subjects for the three gains 20, 50 and 80 euros

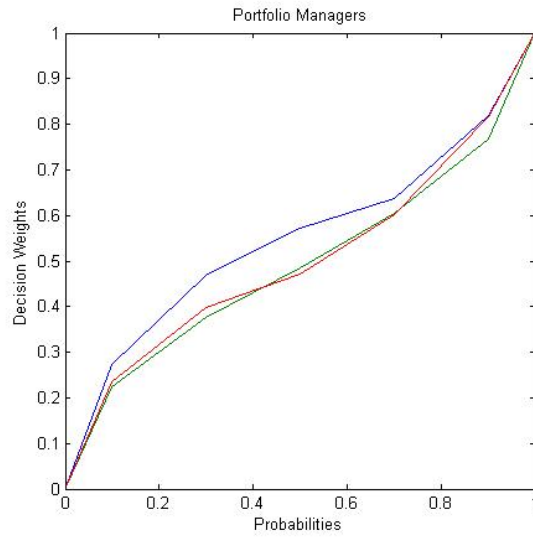


Figure 13: The median probability weighting functions of portfolio managers for the three gains 20, 50 and 80 euros

in the population of portfolio managers, 23 decision weights were bounded up and 23 were bounded down).

The last two figures compare the attitude toward probabilities of both populations for each of the three gains. In the three cases, there seem to be no significant differences between the two populations.

Our results then look very different from those of Fox et al. (1996). How can it be explained? First, the elicitation of the value function was based on the Trade-off method in Fox et al. (1996), whereas I chose not to use data from the Trade-off method and instead to base the elicitation of the value function on the data from the Certainty Equivalent method. Provided the inconsistency between the two methods highlighted in the fourth section, it is not surprising that Fox et al. (1996) obtained linear value functions while we had concave value functions. However, that latter statement does not explain the difference in the elicited median probability weighting function. Indeed, I tried to consider the value function as linear in the computation of the median decision weights and this did not change the degree of distortion of the probability weighting function.

Second, as previously noted, the elicitation of certainty equivalents (as well as values in the Trade-off method) was based on binary choices in one study, while it was directly performed in the other.

The differences could also come from the fact that I only had 15 portfolio managers whereas Fox et al. (1996) had 88 option traders ⁶.

Finally, it might simply be the case that option traders and portfolio managers are two populations who do not behave similarly under risk.

5.5 Interpretation

In the introduction, I suggested that studying the behavior of portfolio managers would provide us with insights about the interpretation that should be given to the concept of attitude toward probabilities. About this topic, I mentioned the literature related to the Discovered Preference Hypothesis by Plott (1996). Let us describe the relation between the present work and this literature so as to delimit the scope of this study.

Experiments showing departures from standard economic models have received a major critic that can be summed up by the following quote from Binmore (1994):

"But how much attention should we pay to experiments that tell us how inexperienced people behave when placed in situations with which they are unfamiliar, and in which the incentives for thinking things through carefully are negligible or absent altogether?"

In this line Plott (1996) provided the so-called Discovered Preference Hypothesis (DHP) which can be explained as follows: subjects have true preferences that satisfy the axioms of standard economic models, but in unfamiliar contexts of decision, they need both incentives and experience about the consequences of their decisions to discover those preferences. In the case of decision under risk, this hypothesis would mean that non-linearities in the treatment of probabilities must be considered as mistakes that will eventually vanish as

⁶Actually 55 option traders and 33 individuals from support staff

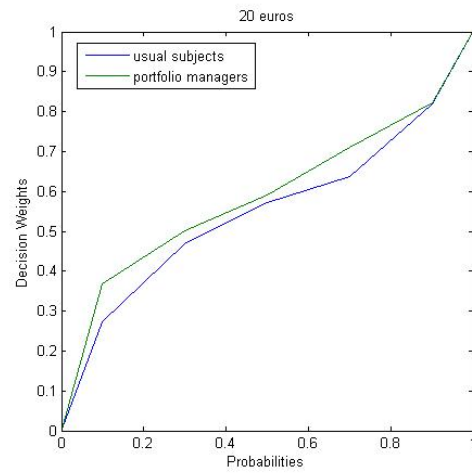


Figure 14: The median probability weighting functions of usual subjects and portfolio managers for the gain 20 euros

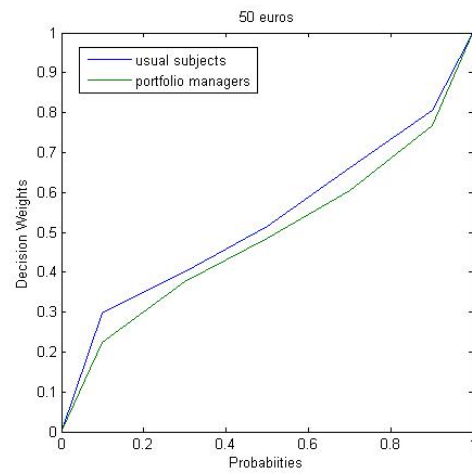


Figure 15: The median probability weighting functions of usual subjects and portfolio managers for the gain 50 euros

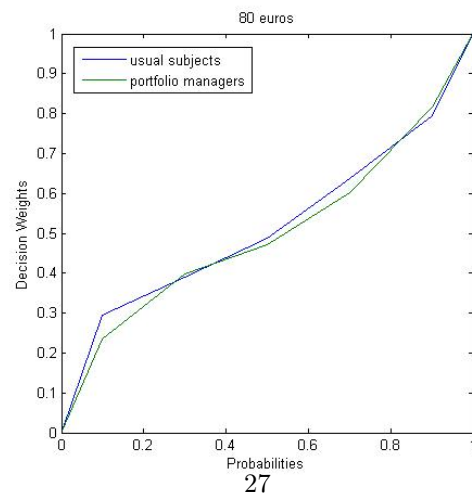


Figure 16: The median probability weighting functions of usual subjects and portfolio managers for the gain 80 euros

subjects get to observe the consequences of their decisions (having enough incentives).

The experiment described in this paper falls into the class of experiments criticized by the literature related to the DPH. Although I think the incentives provided are sufficient (as explained in the third section), the experiment involves only one-shot decisions which means that people do not get to observe the realization of the underlying risk and therefore cannot learn their true preferences. So, the observed probability weighting functions can be interpreted as preferences under risk as well as mistakes that would have disappeared if the experiment involved feedbacks, and we cannot discriminate between those two interpretations.

As a consequence, we shall rather focus on the particular setting of one-shot decisions under risk. This topic of study remains relevant since many economic decisions are one-shot decisions. For example, when one has to decide whether or not to purchase a house, one is definitely deciding in a context without experience about the consequences of his decision.

In this context then, is the attitude toward probabilities a stable concept? Does it vanish as people get a greater level of expertise at dealing with risk or is it a true property of preferences? Our results definitely support the second view. Indeed, the portfolio managers exhibited a greater aptitude at dealing with the experimental task since their answers revealed smaller randomness. In other words, they fitted a model of decision closer than usual subjects of experiments but that model was not the expected utility model. Indeed, each portfolio clearly exhibited a personal attitude toward probabilities that always (except for individual 6) satisfied the usual properties identified in previous works. Furthermore, aggregated data suggest that the median portfolio manager is not closer to the expected utility model than the median usual subject. Therefore, we cannot even say that people with a greater aptitude at dealing with risk show a greater level of neutrality toward probabilities.

The experiment then supports the view that the behavior revealing a distortion of probabilities reveals a robust psychophysical mechanism (in the sense that it is not affected by the experience at dealing with risk) that must be accounted for as soon as one wants to study one-shot decisions under risk.

6 Conclusion and further research

The present work makes two types of contributions: on the methodological side, it proposes a new specification for the probability weighting function, namely the cumulative distribution function of the beta law. The shape of this functional form is governed by its two parameters in a different manner than the existing two-parameter functions. One parameter captures the degree of overweighting of small probabilities while the other one controls the degree of underweighting of large probabilities. This thus allows computing the correlation between those two properties of the attitude toward probabilities. This correlation is up to 80% on our sample.

This paper also introduces a protocol that aims at improving the amount of information that is extracted from one individual in a given number of questions. Its principle is to use prior information about the subject's behavior (obtained at the beginning of the experiment) to calibrate questions. The protocol makes

both the TO and CE methods intervene. Those two techniques of elicitation turned out to be inconsistent.

On the empirical side, the contribution consists in the study of the attitude toward probabilities in the context of one-shot decisions on a population of portfolio managers, that is people having a high level of expertise at handling decision making under risk. It came out that those people were indeed more consistent than people having less experience (like the usual subjects of the LEEP), but this consistency did not made them converge to the expected utility model. Instead, each portfolio managers revealed his own attitude toward probabilities.

This work opens a few interesting paths for future research. First, one of the main lesson that must be retained from this work is how interesting are people whose job consists in making financial decisions under uncertainty (like the portfolio managers). Indeed, they provided us with high quality data in the sense that their choices fit very closely to a model of decision under risk, which renders the analysis much easier. That kind of population is also interesting because it allows seeking for relations between preferences elicited in experiments and some real financial decisions. If one is willing to relate experiments to reality, this must be one of the most promising direction of research.

Another issue raised in this work is the inconsistency between the TO and CE methods. This question should definitely be addressed experimentally at least to check the reliability of the result. If this latter was to be confirmed, then important questions shall be asked about both the job done in experiments and the job done by our models.

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